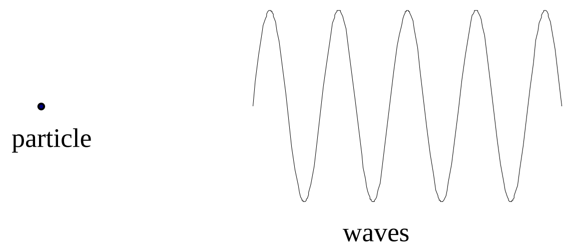
Lecture 1: Classical Physics & The Rise of Quantum Theory

Q: Why learn quantum theory?
Quantum theory gives the correct description of physics at molecular and atomic level or smaller. In fact, it is thought to be the underlying theory of all physical phenomena despite the effectiveness of classical physics for most macroscopic phenomena.

Applications: solid state physics, spectroscopies, modern optics, nuclear & particle physics, superconductivity & superfluidity, cosmology & quantum information theory

The Structure of Classical Physics

Classically: two different entities with two different theories:



- <u>Newtonian mechanics</u> for particles (localised)
- <u>Maxwell's electromagnetic theory</u> for electromagnetic waves* (extended)

*Mechanical waves etc. fall under the class of system of particles.

Newtonian equations of motion:

$$\boxed{\underline{F} = m\ddot{\underline{q}}}$$

The complete solution to this second order differential equation requires two additional information (constants of integration): $(\underline{q}_0, \dot{\underline{q}}_0) \Rightarrow$ state of the physical system.

Concepts of <u>state</u>:

- Minimum information needed to assign values to all observables;
- Minimum information needed to determine states at other times.

Classical (mechanical) state = position-velocity pair or its equivalent.

Mechanics of more general formalism: Lagrangean mechanics or Hamiltonian mechanics.

Lagrangean Mechanics

Lagrangean function $L = L(\underline{q}, \dot{\underline{q}})$ for conservative systems

$$L(\underline{q},\dot{\underline{q}}) = \tfrac{1}{2} m \dot{\underline{q}} \cdot \dot{\underline{q}} - V(\underline{q})$$

Equations of motion = Euler-Lagrange equations
(second order w.r.t. time):

$$\frac{\partial L}{\partial \underline{q}} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\underline{q}}}\right)$$

$$\Rightarrow \qquad -\frac{\partial V}{\partial \underline{q}} = \frac{d}{dt}\left(m\dot{\underline{q}}\right)$$

which is equivalent to Newton's equation of motion.

Lagrangean mechanical state = $(\underline{q}, \dot{\underline{q}})$

Example: Simple harmonic oscillator in one dimension

$$L = \tfrac{1}{2} m \dot{x}^2 - \tfrac{1}{2} k x^2 \qquad \Rightarrow \qquad m\ddot{x} = -kx$$

Example: Electron of charge e between plates of potential Φ_0 and distance d.

$$L = \frac{1}{2} m(\dot{\rho}^2 + \rho^2\dot{\phi}^2 + \dot{z}^2) - \frac{e\Phi_0}{d} z$$

$$\Rightarrow \begin{cases} m\ddot{\rho} = m\rho\dot{\phi}^2 \\ m\rho^2\ddot{\phi} = 0 \\ m\ddot{z} = -e\Phi_0/d \end{cases} .$$

Hamiltonian Mechanics

Hamiltonian function $H = H(\underline{q}, \underline{p})$ for conservative system

$$H = \frac{\underline{p} \cdot \underline{p}}{2m} + V(\underline{q})$$

where $\underline{p} = \frac{\partial L}{\partial \dot{\underline{q}}}$ = conjugate momentum.

Hamiltonian mechanical state = $(\underline{q}, \underline{p})$.

Equations of motion = Hamilton's equations:

$$\frac{d\underline{q}}{dt} = \frac{\partial H}{\partial \underline{p}} \quad ; \quad \frac{d\underline{p}}{dt} = -\frac{\partial H}{\partial \underline{q}}$$

Example: Electron of charge e between plates of potential difference Φ_0 and distance d.

Conjugate momentum: $p_\rho = m\dot{\rho}$; $p_\phi = m\rho^2\dot{\phi}$; $p_z = m\dot{z}$

Hamiltonian: $$H = \frac{p_\rho^2}{2m} + \frac{p_\phi^2}{2m\rho^2} + \frac{p_z^2}{2m} + \frac{e\Phi_0}{d} z$$

Equations of motion:

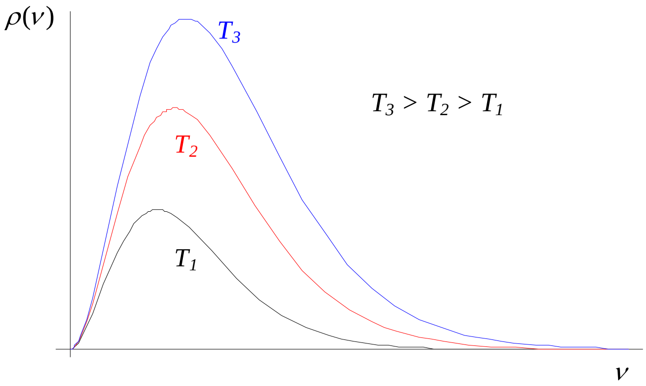
$$\dot{\rho} = \frac{p_\rho}{m} \; ; \quad \dot{\phi} = \frac{p_\phi}{m\rho^2} \; ; \quad \dot{z} = \frac{p_z}{m} \; .$$

$$\dot{p}_\rho = \frac{p_\phi^2}{m\rho^3} \; ; \quad \dot{p}_\phi = 0 \; ; \quad \dot{p}_z = -\frac{e\Phi_0}{d} \; .$$

The Failure of Classical Physics

Blackbody Radiation

Black body – ideal body that absorbs all incident radiation. At sufficiently high temperature, the body reradiates e-m radiation – producing *blackbody spectrum* with energy density ρ with temperature T dependence:



Classical physics: Rayleigh-Jeans distribution

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} kT$$

– diverges with frequency ν and temperature T – *ultraviolet catastrophe.*

Other distributions: Wien distribution – empirical in nature

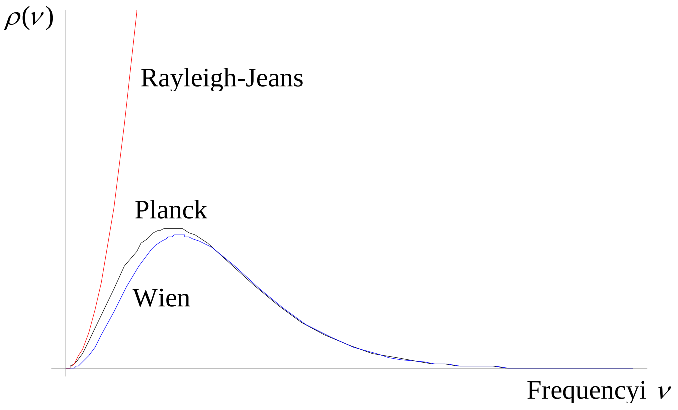
$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} e^{-h\nu/kT}$$

– agrees with experiment at high frequencies.

Planck distribution

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$

– agrees with experiment!



Planck's explanation: Quantity *(quantum)* of energy being absorbed or radiated by body is in discrete form:

$$E = nh\nu \qquad (n \in \mathbb{Z}^+)$$

Planck's constant

$$h = 6.626176 \times 10^{-34} \text{ Js}$$

Photoelectric Effect

Phenomena of radiating photoelectrons when e-m radiation hits surface of metals– photoelectric effect.

Milikan:

- Number of electrons N_e increases with increasing e-m radiation intensity I;
- Energy of electrons E_e increases with frequency ν of e-m radiation.

Classical physics:

- When intensity I increases, the energy E_e increases;
- Frequency ν has no direct relation with N_e or E_e.

Einstein:

- E-m radiation consists of discrete quanta of energy $h\nu$ – the 'particle' photon (given the symbol γ)
- On increasing ν, the photon's energy E_γ transferred to the electron is higher (higher allowed maximum energy of photoelectrons).

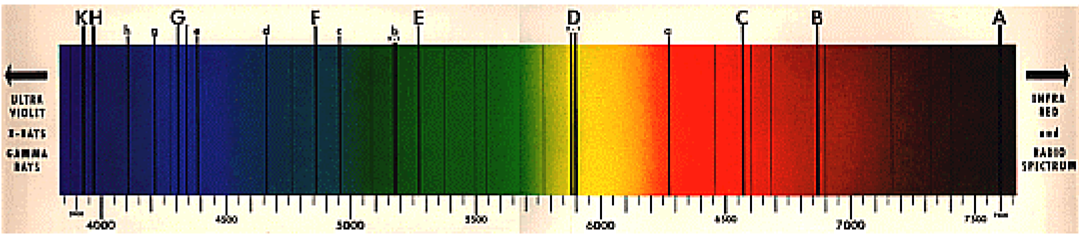
- On increasing I, there will be more number of photons N_γ colliding with electrons (maximum energy of photoelectrons stays the same).

The particle nature of e-m radiation is also verified in the experiment of Compton effect involving scattering of x-ray photons by electrons.

Atomic Spectrum

Much earlier: all elements produce unique line spectrum – underlying various spectroscopies.

The concept of line spectrum has no explanation – there is no quantitative theory of atoms.



(Source: http://www.coseti.org/solatype.htm)

There exists an empirical formula for the frequencies of the line spectrum e.g. *Balmer formula* for Balmer series of hydrogen atom:

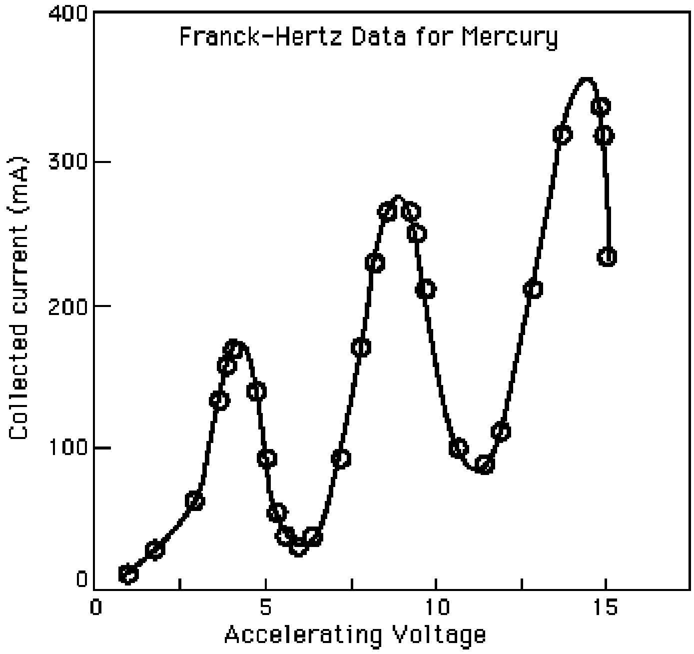
$$\lambda_n = 364.56 \frac{n^2}{(n^2 - 4)} \text{ nm} \quad ; \qquad n = 3, 4, \cdots$$

More general: Rydberg-Ritz formula

$$\frac{1}{\lambda_{nm}} = \frac{\nu_{nm}}{c} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad ; \qquad (n \geq m\,,\ m > 0)$$

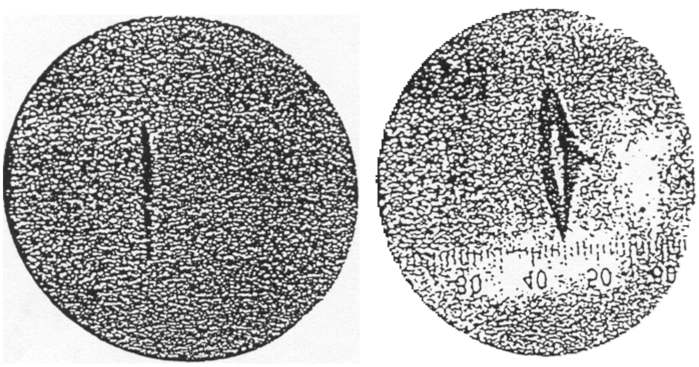
The constant R = Rydberg constant (its value depends on the element).

Franck-Hertz experiment shows the line spectra phenomena also occurs in matter-matter interaction (electrons and mercury atoms).



(Source: http://hyperphysics.phy-astr.gsu.edu/hbase/FrHz.html)

Stern-Gerlach Experiment shows that the magnetic moments of atoms do not have a continuous distribution but instead show quantized directions – giving rise to the concept of *spin*.



(Source: http://setis.library.usyd.edu.au/stanford/archives/fall1999/entries/physics-experiment/figure13.html)